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SELECTION OF APPLICABILITY LIMITS OF A MATHEMATICAL MODEL OF TURBULENCE IN FORMATION OF A STEEL INGOT

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Rational applicability limits of a turbulent model of momentum and heat transfer in a solidifying melt in an enclosure are determined based on multivariant calculations of hydrodynamic and thermophysical processes during casting and solidification of steel ingots.

Hydrodynamic processes determine the thermal and physicochemical situations in a solidifying steel ingot. Calculation of velocity fields is a laborious problem requiring substantial computer time. Thus, calculation of velocity fields in a liquid core of a crystallizing melt takes 60% of computer time, and the remaining 40% is required for determination of the temperature field and the concentration and amount of the solid phase. If the process has a turbulent character, then 62–80% of computer time is spent in calculation of the hydrodynamic part of the problem. The percentage depends on the selected model of turbulence closure. Therefore, the choice of a mathematical model is a complex process: on the one hand, neglect of turbulent transfer of substances can lead to considerable distortion of results, and on the other hand, unsubstantiated allowance for turbulence considerably increases the time of computer calculation.

This paper is aimed at determination of the rational applicability limits of a turbulent model of momentum and heat transfer in a solidifying melt in an enclosure, since for enclosures this problem is still unsolved.

Experimental measurements in high-temperature melts are difficult; therefore, mathematical simulation of hydrodynamic processes and of heat and mass transfer using the k-ε model of turbulence in casting and solidification of a steel ingot is chosen as the method for investigation [1-3]. The adequacy of the mathematical model was confirmed earlier [4] in modeling of processes of momentum, heat, and mass transfer in solidifying ingots.

The studies were conducted for two stages of ingot formation: casting into the mould and solidification. There is an alternative to the first stage, viz., to calculate the hydrodynamics in either a laminar or turbulent approximation. Figure 1 presents the dependence of the thickness of the hard skin averaged over the ingot height on the ingot tonnage in casting from above and through a bottom gate (from below). The effect of the casting method on the behavior of the melt was estimated for equal technological parameters (superheating and velocity of melt inflow to the mould) both in casting from above and through a bottom gate.

In casting from above, starting from a 1-ton ingot, allowance for turbulence noticeably affects the thickness of the hard skin. An increase in mass intensifies these differences. In casting through a bottom gate the effect of turbulence manifests itself starting from an 8-ton ingot (Fig. 1).

The difference in melt behavior, depending on the method of casting, is explained as follows. During casting the intensity of melt mixing is characterized by two factors: the mechanical action of the jet and heat convection. They govern the development of mixed convection in a melt, which determines the hydrodynamic situation in the melt and, as consequence, heat and mass transfer. In casting from above there are two zones. The first exists during the entire process of casting and is characterized by the presence of descending flows in the center of the mould that are caused by jet divergence. As the mould becomes filled (50%) the second zone forms, in which a reverse flow is observed, i.e., in the center there are ascending flows and in the near-wall region descending flows occur

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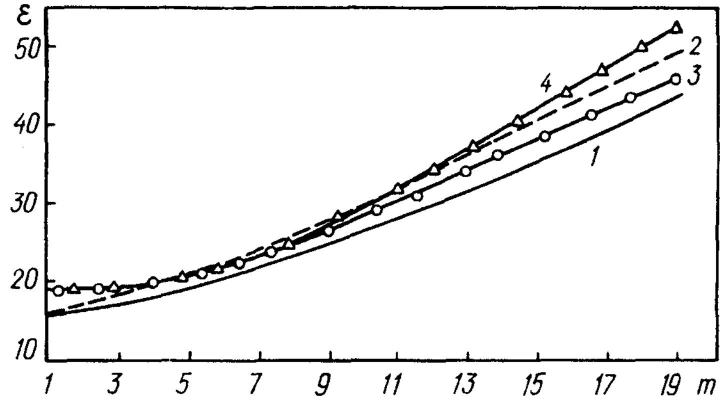

Fig. 1. Dependence of hard-skin thickness averaged over ingot height on ingot tonnage in casting from above (without (1) and with (2) allowance for turbulence) and in casting from below (without (3) and with (4) allowance for turbulence). ε, mm; m, ton.

over the entire height of the mould [1]. This zone is caused, first of all, by a thermal gradient. The intensity of agitation in the second zone is by an order of magnitude smaller than in the first zone. As the region becomes filled the dimensions of the first zone do not change, and the second zone occupies an even larger volume.

Eddy viscosity ν_t substantially affects the thickness of the hard skin over the ingot height. By preventing jet propagation inside the melt [2], it facilitates the early formation of the second, more quiet, zone. While on the upper horizons the jet-induced flows transfer heat, thus melting the hard-skin, on the lower horizons descending flows take heat from near-wall layers and carry it away to the ingot center, thus facilitating hard skin growth.

In casting through a bottom gate, we observe three flow zones: central, represented by ascending flows caused by jet divergence; peripheral, formed by descending flows induced by jet ejection and heat convection; and a zone of ascending flows localized at the upper angle, which is formed by the melt level and the side wall. As in casting from above, the highest velocities (up to 60 cm/sec at a flow rate of 1000 kg/min) occur in the first zone. However, while in casting from above this zone moves as the mould becomes filled, in casting through a bottom gate it is motionless. The height of the jet plume does not exceed 70–100 cm, and eddy viscosity has practically no effect on its height.

The second zone is more quiet (maximum velocity attains 20 cm/sec) and localizes in the bottom portion of the mould. At the beginning of filling, melt agitation is caused by jet ejection. With the loss of heat equilibrium in the melt, the intensity of mixed convection increases due to convective heat transfer. It should be noted that the effect of turbulence is higher in the second zone. This is manifested by a more intense hard-skin growth. Apparently, ascending flows, having transferred heat at a height of 1–1.2 m, become cooled and descend to the mould bottom, thus facilitating hard-skin growth. It is clear that with turbulent heat removal the process of heat transfer is more intense.

The third zone is the most quiet (maximum velocity is 1.5–2 cm/sec); the jet is not active here and temperature, which is uniformly distributed over the ingot cross-section, does not promote heat convection. This zone is formed after 50% filling of the mould; therefore, the hard skin is the thinnest. The effect of turbulence virtually is not expressed.

With an increase in the mould dimensions the greatest effect is exerted by the mould width: growth of the second zone and of the thickness of the hard skin is observed.

It is shown by the examples of two types of casting that the value of eddy viscosity makes substantial ammendments to the thermal and hydrodynamic situations. Therefore, to estimate the rational applicability limits of the turbulent model it is suggested to use similarity criteria Re and Gr_*: the first characterizes the outer situation, which is determined by the jet, and the second characterizes inner situation caused by natural heat convection. It makes sense to introduce the complex $ReGr_*$, where $Re = Vd/\nu$, $Gr_* = g\beta\Delta T x_{red}^3/\nu^2$. The choice of x_{red} makes it possible to allow for different geometric ratios of the ingot [5].

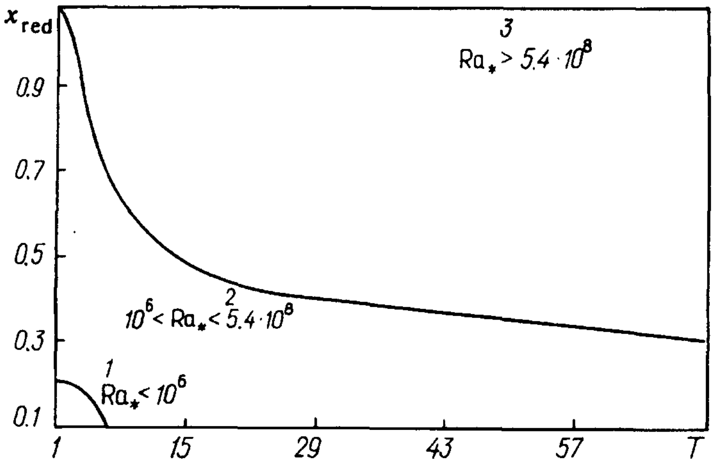

Fig. 2. Diagram for selection of a mathematical model depending on reduced size and melt superheating: 1) nonconvective, 2) laminar convection, 3) turbulent models. x_{red}, m; T, °C.

An analysis of the results of the computational experiment (CE) showed that allowance for turbulence is necessary at the following values of $ReGr_*$: $8.9 \cdot 10^{12}$ for casting from above and $3.8 \cdot 10^{13}$ for casting through a bottom gate.

The second stage of investigations is the determination of applicability limits of the turbulent model in ingot solidification. Here, due to cessation of casting the Rayleigh number $Ra_* = PrGr_*$ is a determining criterion.

Comparison of solidification times of a 1-ton ingot calculated in the absence of convection and with natural melt mixing showed that the effect of hydrodynamics in a vertical ingot can be ignored when $Ra_* < 8.2 \cdot 10^5$. But for such Ra_* ($Pr = 0.216$ and $x_{red} = 0.095$ m) superheating should not exceed 1°C, which is technologically unlikely in ingot casting. Therefore, it can be stated that convection should be taken into account in solidification of steel ingots with a mass larger than 1 ton.

As for allowance for turbulence, twelve versions of solidification of blooming ingots with weights of 2.7, 5.75, 10, 12, 18, 20, 24, 42, 98, 120, 380, and 420 tons with superheating $\Delta T = 60$°C were calculated to estimate its effect. A difference in the total time of ingot solidification exceeding 10% starts with a 20-ton ingot with x_{red} = 0.3 m and $Ra_* = 5.1 \cdot 10^8$. For a 420-ton ingot ($x_{red} = 0.83$, $Ra_* = 1.1 \cdot 10^{10}$) these differences amount to 19%. Based on the calculations a diagram was constructed that makes it possible to select a corresponding mathematical model depending on x_{red} and ΔT (Fig. 2).

When calculating one version of steel ingot solification one can successively use several models: turbulent, laminar and nonconvective. For a 20-ton ingot cast at $\Delta T = 60$°C one should use a model allowing for turbulence, since, according to the diagram, the value of Ra_* is found in region 3 corresponding to a turbulent model. At 17.5 min after the start of solidification due to cessation of superheating [6] the values of Ra_* allow one to speak about the possibility of laminar model employment and after 87 min one can ignore convection. The analysis made if possible to considerably reduce computer time: 29 h of IBM 286/287 computer time were spent using the turbulent model during the entire time of 20-ton ingot solidification, and 13 h in combined use of different models (Fig. 2). In this case the difference in the time of ingot slidification was 2.3%.

Thus, the total time of ingot solidification t_0 is conventionally represented in the form

$$t_0 = t_t + t_{lam} + t_{nonconv} = t_{conv} + t_{nonconv} \,. \tag{1}$$

To determine the dimensionless time of employment of the convective model we processed the results of calculation of solidification for the 12 above-mentioned versions. On the basis of statistical processing of the results the following formula was obtained

$$Fo_{conv} = 5.39 \, Ra^{-0.15} \,. \tag{2}$$

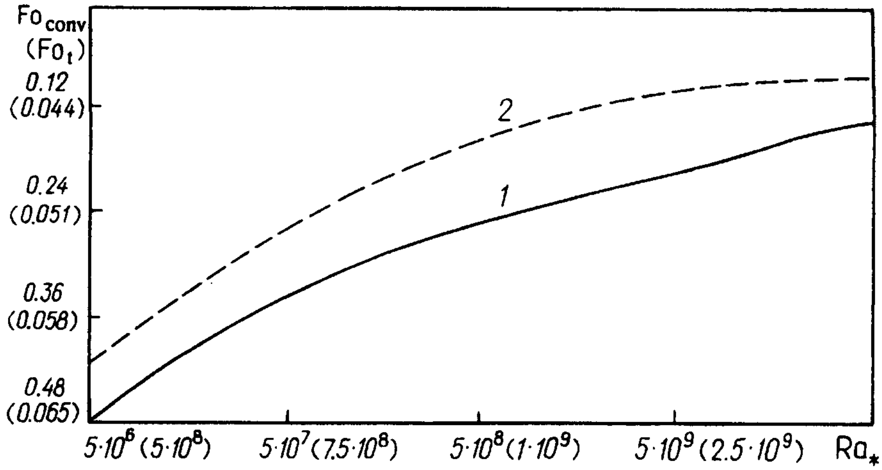

Fig. 3. Dimensionless times of existence of convection (curve 1) and turbulent convection (curve 2; the values of the parameters are given in brackets at the abscissa axis and the axis of ordinates) as functions of Ra_*.

For example, for a 24-ton ingot made of carbon steel St08 (x_{red} = 0.32 m and ΔT = 60°C) $Ra_* = 6.1\cdot10^8$. Substituting this value of Ra_* into Eq. (2), we obtain $Fo_{conv} = 5.25\cdot10^{-2}$. We find the dimensional time at which it is recommended to allow for the effect of natural convection: $t_{conv} = Fo_{conv}x_{red}/a$, and for the steel mentioned it equals 97.1 min. Further allowance for convection is not expedient, since, due to the small rates of mixing, it does not affect the total time of ingot solidification or the shape of the hard skin [6].

To determine the time range of the existence of convection in a turbulent regime we conducted studies for 20-, 24-, 42-, 98-, 120-, 380-, and 420-ton ingots at ΔT = 60°C and constructed the dependence (Fig. 3, curve 2)

$$Fo_t = 0.208\, Ra_*^{-0.068}\,. \qquad (3)$$

The technique for determining the rational applicability limits of a turbulent model in calculations of hydrodynamic and thermophysical processes in an ingot is similar to that presented above for allowance for a convective model. For a 24-ton ingot the time of the existence of turbulent convection is t_t = 19.6 min. Thus, to calculate hydrodynamic and thermophysical processes in a 24-ton ingot made of carbon steel St08 it is reasonable to use the turbulence model during first 19.6 min of ingot solidification. Then, at $t_{conv} - t_t$ = 77.4 min a laminar convective model should be used.

CONCLUSIONS

1. Rational applicability limits of turbulence are found for casting from above and through a bottom gate: allowance for turbulence is necessary for casting from above when $ReGr_* > 8.9\cdot10^{12}$ and for casting through a bottom gate when $ReGr_* > 3.8\cdot10^{13}$.

2. Allowance for laminar convection is needed when $Ra_* > 1\cdot10^6$, and for turbulent convection when $Ra_* > 5\cdot10^8$. A diagram for selection of turbulent, laminar, and nonconvective models of ingot solidification is constructed.

3. The time of the intense effect of natural laminar and turbulent convective heat transfer on hydrodynamic and heat and mass transfer processes in a solidifying ingot is determined. Dimensionless relations for determining the rational time for employing laminar and turbulent models of convection are obtained depending on the main technological parameters of the ingot.

NOTATION

ν_t, eddy viscosity; Re, Gr_*, Reynolds and Grashof numbers; V, velocity of metal inflow to mould; d, jet diameter near metal surface durring casting from above or in sleeve during casting from below; ν, melt viscosity; β, coefficient of metal volumetric expansion; $\Delta T = T_0 - T_L$, thermal superheating, difference between casting temperature and liquidus; x_{red}, reduced size of ingot (ratio of volume of ingot to its surface area); $Pr = \nu / a$, Prandtl number; a, thermal diffusivity; t_t and t_{lam}, times of existence of turbulent and laminar convection; t_{conv}, total time of existence of convection in liquid core of solidifying melt; $t_{nonconv}$, time of nonconvective ingot solidification; $Fo_{conv} = at_{conv}/x_{red}^2$ and $Fo_t = at_t/x_{red}^2$, dimensionless time of employment of convective and turbulent models.

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